A Simple Complete Conservative Sustainable Agent-Based Model Economy – Some Hypotheses

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Abstract. With the recent and wide interest in sustainable development, especially among those in the younger generations, a project was undertaken to identify the necessary and sufficient conditions for a sustainable agent-based model economy. We started with a baseline system composed of two interacting subsystems. At the biophysical level farmers and workers exist in a world constrained by the conservation of mass and energy, and in which the agents compete for life-sustaining resources. At the economic level each transfer of mass and energy must be reciprocated with a transfer of cash. Features were added to the baseline model until sustainability was achieved. This model was then analyzed by the author for its interesting characteristics with respect to social structure and governance. In conclusion, the author tables a set of hypotheses about the nature of sustainable model economies to guide further studies.

1   Introduction

The software application that plays a key role in this paper, called ModEco, is designed with the intent to enable students of sustainable economies to configure a model agent-based economy on a desktop computer, and see the economy develop in minutes as they watch. Inspired by a thought-provoking book called "The Ecology of Commerce" by Paul Hawken [1], it was developed in conjunction with an enrichment program for exceptionally gifted students in an Ontario high-school. The ultimate goal in the development of the ModEco application was to identify the necessary and sufficient components of a most simple but complete sustainable economy, and look for policy implications. That goal has still only been partially realized.

We started with a previously studied fully sustainable biophysical model called PSoup in which bugs live and evolve in a pond. We modified the bugs to become farmers and workers; we modified the activities performed in each tick to emulate commercial activity; we added the exchange of cash for goods and services; and so with a number of design changes produced a baseline model of a human economy.

Although the program functioned without significant error, the economy always crashed almost immediately, with all agents dying. That started a process of adding functions and features in a search for that combination of such that would produce a sustainable economy.

2   The Baseline Model – An Overview

The ModEco application is written in C++ with MFC using the Microsoft Visual Studio 2010 application development environment. The standard libraries have been augmented with an implementation of the Mersenne Twister, a pseudo-random number generator (PRNG) which provides more substantive control over stochastic experiments than is available through the built-in PRNG. [2-3] The compiled ModEco program will work on Windows XP or Windows 7 systems as a standalone app requiring no additional runtime libraries. The compiled program, a C++ workspace, a high-level design document, and a 39-page ODD model description are available from the model library at [4]. In addition, various compiled versions of ModEco and PSoup, together with informal analyses and reports, are available at [5]. Space does not permit a complete description of the model here, so the following is a summary.

A ModEco-based economy is a merger of two closely linked dynamic subsystems fashioned with a very simple post-barter agricultural economy in mind. An economic subsystem rides atop a biophysical subsystem. The agents come in two varieties playing two different biophysical/economic roles: farmers and workers.

At the biophysical level, farmers and workers work together to harvest grain from fields and store it in the farmers' inventory bins, carry the grain home and place it in their pantries, eat it, and pass the waste mass back to the farmers to put back on the fields to start the cycle over again. Agents' lives are controlled by biophysical parameters that control things like food consumption rates, reproductive maturity, or death by starvation or old age. Agents compete for life-sustaining resources, and access to resources is mediated by genetic code. But, farmers and workers are co-dependent. If they do not work together to harvest food, none is harvested and they all die. All farmers have the same strategy. All workers have the same strategy. These agents must compete for their share of the wealth or die, but but farmers and workers also co-operate.

Tightly intertwined with the biophysical subsystem is the economic subsystem. Farmers hire workers to work with them in the fields to harvest the grain. They then sell the grain to consumers. The consumers sell waste mass to the farmers who spread it on their fields. A single good is bought and sold in its various forms (recycled waste, fresh grain for sale, supplies for consumption, waste). A single service is bought and sold (harvesting grain).

Together, these two logical subsystems are so tightly linked that they form a single dynamic system having both biophysical and economic characteristics. Within this system all mass and energy flows in one direction, mediated by transfers of cash in the opposite direction. This economy is described as "complete" because (a) it includes both biophysical and economic dynamic subsystems; and (b) it models a com-

plete cycle of mass and energy as it flows through both subsystems, from farmer's field, through consumers and back to farmer's field. It is described as "most simple" because it is difficult to conceive of an economy that is more simple, but still complete in the sense described above.

In ModEco the action happens in a rectangular area called the township, which is tiled by squares called lots. A lot may be undeveloped, a farm lot housing one farmer, or a residential lot housing from one to four workers. Within one tick an agent may directly affect other agents or lots within a neighbourhood of 25 lots called its commuting area. Except for dealings with immortal agents (described below), agents may only move within or do commercial business within their own commuting areas. Each agent has limited knowledge of the economy based only on its own successful transactions within its own commuting area.

Value in ModEco is determined by metabolic requirements. Agents require a steady diet of mass and energy to live, and failure to eat a sufficient amount of each on any one tick causes instant death by hunger. Food contains mass and energy which, on consumption, become waste mass and metabolic energy. Units of mass and energy are each assigned intrinsic value which is unchanging. Four quantities are conserved on a transaction-by-transaction basis, and also at a system-wide level: cash, mass, energy, and, by implication, intrinsic value. These quantities must cycle through the economy continuously or it will collapse, with the death of all agents. Net worth is the sum of the intrinsic value plus cash, and it is conserved at the system level.

Time is measured in discrete units called ticks. In one tick of the ModEco clock the following ten functions happen in order: (a) setup; (b) make job offers; (c) move workers; (d) sell inventory; (e) consume supplies; (f) sell waste; (g) buy recycled waste; (h) reproduce agents; (i) death of agents; and (j) cleanup. Together, these ten biophysical/economic functions are called the 'economic engine', and it is executed once per tick of the ModEco clock.

Agents maintain valued quantities in stores called asset classes. Farmers each have six asset classes in stores of cash, energy, recycled mass, an inventory of food for sale, a supply of food for personal consumption, and waste mass. Workers each have four asset classes in stores of cash, energy, a supply of food, and waste mass.

Ideally a sustainable computer-based model economy would run forever without population collapse, as was easily achieved with a purely biophysical agent-based model in PSoup. More pragmatically, a model economy is designated sustainable, somewhat arbitrarily, if it will run for one million cycles without collapse, thereby providing a substantial volume of data for analysis.

Regrettably, to date, a simple sustainable ModEco-based economy called the PMM has been demonstrated only under very severe constraints. The intellectual journey that has led to the successful design of this sustainable model economy has been difficult, and full of unexpected insight. The purpose of this article is to briefly describe the features added to achieve sustainability, share insights derived from analysis of that sustainable society, and to lay out a course for further study.

3   The Search for Feature Sufficiency

The parameter space of a ModEco-based economy has various dimensions, many of which come from the elements of the feature set that was added to the basic bio-physical/economic model in the search for sustainability. When a sustainable economy was ultimately achieved, it was with a mix of such features. The baseline economy plus those added features that are enabled in the PMM can be considered the list of features sufficient to demonstrate a sustainable model economy.

Added Feature #1 – Process Quotas. The economy fails if agents spend all of their money at the first opportunity. Large purchases cause two problems: (a) other agents have reduced access to needed resources, and they die; and (b) the wealthy agents concentrate too much of their wealth in one asset class and so have restricted ability to participate in the economy, and they die. For example, if a farmer spends all its money hiring workers, it may have insufficient money to purchase supplies for its own consumption.

Added Feature #2 – Municipal Grants and the Estate Manager. The economy fails if the assets of dying agents are not redistributed to the living agents. The Estate Manager (EMgr) is an immortal agent with no fixed locale that has access to every agent on every tick. Any agent that has successfully negotiated a commercial transaction but has insufficient quantities of an asset to meet process quota may apply to the EMgr for a municipal grant, up to quota. The primary role of the EMgr is to provide a means for all estate assets to be returned to the economy.

Added Feature #3 – Business Factors. The economy fails if agents do not maintain a practical distribution of their wealth across key asset classes. For example, the "Buy Supplies Factor" (BSF) is set to 0.25 for workers. A worker will not purchase supplies unless the value of his supplies drops below 25% of his total net. These business factors trigger the participation of the agent in any commercial opportunity. Without such a restraint, the poorer agents are apt to put all of their assets into the wrong asset class, and they die.

Added Feature #4 – Waste Recycling and the Materiel Manager. The economy fails if all of the waste mass accumulates in the middle of the community. In the baseline system, consumers sold their wastes directly to farmers. The farmers with many consumers within their commuting areas prospered, those at the edges of the community did not, and the waste and recycled mass concentrated in the centre of the community. An immortal agent called the Materiel Manager (MMgr), having no fixed locale, and having equal access to all consumers and all farmers, was given the job of buying waste from consumers and selling the recycled mass to farmers.

Added Feature #5 – Municipal Deficit. The economy fails if the amount of cash is not correctly scaled to the amount of mass and energy, but the correct scaling is difficult to determine if a sustainable economy has not yet survived to steady-state. To resolve this issue, the MMgr was given the ability to spend money which it did not have when purchasing waste from consumers. It was allowed to go into debt, and when steady-state was achieved, the correct scaling was determined empirically. However, the scaling varies when other parameters are adjusted. For robust sustainability, this feature seems very useful, and even necessary.

Added Feature #6 – Price Controls. However, with all of the above added features, the baseline system was still able to function for only a few thousand ticks before all agents perished. Typically inflation or deflation became so severe as to cause economic failure, and the death of all agents.

In the baseline system, agents negotiate prices, and in every transaction there is a small potential gap between the value of the cash changing hands and the value of the goods. This price/value gap causes the net worth of one participating agent to rise, and the net worth of the other agent to fall. Over time, price changes suffer a positive feedback mechanism and they spiral upwards or downwards out of control. Elimination of this price/value gap resolved this problem, and the first and only sustainable configuration was found. Note that this implies that (a) each agent knows the precise intrinsic value, in metabolic terms, of the goods and services being sold, and (b) prices are fixed at that intrinsic value.

The Search for Sustainability Ends – The PMM. There is a similarity between the role of friction as it leaks energy out of a spinning top and the price/value gap as it leaks value out of an economy. With such an image in mind, we dubbed the perpetually running economy the Perpetual Motion Machine (PMM). In the PMM there are quotas limiting the size of all commercial transactions, prices are precisely controlled, gene-based variations in agent abilities are neutralized, and central authorities redistribute wealth according to public policy. Clearly the PMM is an extremely constrained idealization of what most people would envisage as a "sustainable economy". Even relative to the range of features available in ModEco, the PMM is a feature-poor economy. However, to date, all variations from this constrained model quickly collapse. The PMM, on the other hand, has run to an amazing twenty million ticks. The fact that the PMM is the only sustainable model economy achieved so far makes it of particular interest, and the focus of the rest of this article.

Such a PMM is conservative of the net worth of its agents in every transaction. There is no room for profit and loss on any transaction. And it requires a level of knowledge that is unavailable to people in a real-world economy. Who among us knows the intrinsic value of a day's labor, in joules or in dollars? However, this highly constrained "most-simple" agricultural economy is the only sustainable economy produced so far. And, with the inclusion of such mechanisms as process quotas and business factors for the agents, and immortal central agents, the phrase "most-simple"

can certainly be questioned. The apparently necessary complexity of our most-simple model is one of the largest surprises of this ModEco sustainability experiment.

4 Insights From the Search for the PMM

The quest for an agent-based model of a complete most-simple sustainable economy was started with the admittedly arrogant thought "Everybody understands sustainability. There are organizations all over the world talking about it. A simple model will be easy to make. A few farmers; a few workers; buying and selling food; how hard can it be?" Well …

Lack of Scientific Study of Sustainable Economic Dynamics. There seems to be a shortage of scientific studies of the dynamics of complete sustainable economies. Or, at least, there is a shortage of studies which might provide guidance and insight for the creation of a sustainable agent-based model economy in ModEco. There is a need for a focus on the system dynamics of sustainable economies, and that need does not seem to be addressed by any research being done to date. What are the necessary and sufficient conditions for sustainability? What are the emergent operational characteristics of such an economy, the "stylized facts"? What are the policy implications if we wish to achieve some measure of sustainability in our real-world global economy?

Lack of Examples of Sustainable Agent-based Economic Models. Agent-based modeling is a relatively new and rapidly growing method of study of economic systems, strongly recommended by Boulanger and Bréchet for studies of sustainable development. [6] But it was discouraging to discover that no guidance was to be found with respect to how to construct a sustainable model society. Admittedly, the search was not exhaustive, but you would expect that at least one such model would be easy to find. In fact, it was disappointing to discover that there do not appear to be any agent-based models of complete economies, whether sustainable or not.

The Impact of a Price/Value Gap. One of the insights derived more directly from ModEco is the friction-like role of the price/value gap. In the search for sustainability a variety of constraints were built into ModEco, such as quotas on flow rates, and controls on the distribution of assets across asset classes of each agent. Flexibility in population sizes and flexibility in amounts of cash available in the economy were also built in. Nevertheless, with all of these constraints and flexibilities, the economies were still unstable and ungovernable, collapsing due to inflation or deflation within a few thousand ticks leaving all agents dead. The last and most important constraint added was the elimination of the price/value gap in every transaction.

All inflation or deflation of prices comes from the difference between the value of dollars paid and the intrinsic value received, called the price/value gap. Agents negotiate prices for each commercial transaction. If and only if all prices paid are at exact-

ly the intrinsic value, inflation or deflation cannot occur, and the economy will run forever.

We have, then, this perplexing observation. It is difficult to imagine a market-based economy without profit and loss. On the other hand, it has proven very difficult to model a sustainable economy which includes profit and loss.

Sustainability versus Small Government. The two immortal agents (the MMgr and the EMgr) do not die, or reproduce, nor do they have biophysical requirements, and, as such, can be viewed as a metaphor of a township bureaucracy. They are necessarily non-localized, and their actions are not limited to any commuting area nor to any number of agents per tick. That is, they can address the needs of all mortal agents in every tick. They complete the mass cycle in a closed mass system in such a way that the flow of mass is not hampered by two-dimensional access issues. They provide a flexible money supply. They execute a public policy for the redistribution of wealth of dead agents.

Some conservative and/or libertarian ideals value small government and minimal regulation. The PMM is, on the other hand, a highly regulated society in which quotas are applied to every commercial transaction, prices are strictly controlled, scarce resources (waste mass) are managed by central authorities, and wealth is redistributed by public policy with sustainability goals in mind. It would seem that sustainability requires strong government intervention.

Sustainability versus Social Justice. However, there is a radically more disturbing observation. In almost all real-world organizations devoted to sustainability, the goals are closely aligned with goals of social or economic justice. Like motherhood and apple pie, these two utopian dreams are both warmly desired and inseparably inter-twined in the language of most modern ecological activists. A sustainable society is one which shares the wealth fairly among present and future generations. A just society is one which shares the wealth fairly among members of the present generation. It is easy to see how the two themes are so closely bound together in people's minds.

In addition, most modern advocates of social justice hope to see a future in which the standard of living is substantially above the hard-scrabble subsistence-level existence that we see in many third-world poverty-stricken countries today. The dream of social justice is not meant to be reduced to merely an equal sharing of misery and extreme poverty.

Unfortunately, in the PMM, these two ideal goals of sustainability and social justice are utterly at cross purposes with each other. In the PMM, a sustainable economy, there is no social justice. Life is miserable and short for the majority of the population: (a) fifty percent of every generation dies of starvation; (b) of the fifty percent of the population that survives, the bulk of them are desperately poor; and (c) poor agents have radically reduced opportunities to participate in the economy. Those born to wealth get richer, and those born to poverty starve while still young. In even this most simple ModEco-based economy, it is difficult to imagine a way to merge the goals of sustainability and social justice without seriously compromising both ideals.

The Importance of an Understanding of Entropy. Having achieved a sustainable economy in the form of the PMM, data was collected for over one million agents and the distribution of wealth was plotted. It was surprising to see a distribution looking amazingly similar to the Maxwellian distribution of speeds of atoms in an ideal gas – the distribution first worked out by James Clerk Maxwell in 1859, and studied by the author in the late 1960s in physics classes. [7] A little research turned up the paper by Victor Yakovenko in which he explored the "Statistical Mechanics of Money" and the occurrence of such distributions that cross the boundaries from physics to economics. [8] This led me to the new heterodox branch of economics that calls itself Econophysics, and from there Ecological Economics [9-10], and from there to Biophysical Economics [11], [14].

Energy can be used to do work, but it is nevertheless conserved. As that work is done, the energy is said to degrade from useful energy to waste heat. As the energy degrades in usefulness, the entropy of the system increases. The second law of thermodynamics says that, in any closed thermodynamic system, the system will automatically reconfigure itself until entropy is at a maximum (my paraphrase). As the system reconfigures itself, the energy is "used" to "do work", the energy redistributes itself, and the entropy rises. This second law of thermodynamics is sometimes referred to as the Maximum Entropy Principle, or the MEP. When a closed insulated box containing an ideal gas is allowed to reach a state of thermodynamic equilibrium, then entropy within the box will be maximized, and the distribution of the speeds of the atoms in the box will conform to the Maxwellian distribution. So, the appearance of this Maxwellian distribution in the PMM might be viewed as an indicator of the MEP at work in the PMM.

However, one has to ask, how can that happen in a non-physical computer model of wealth? A distribution of wealth is not a purely physical distribution like a distribution of speeds. And furthermore, a distribution of logical values generated by an agent-based model of wealth is even further removed from physical reality. A recent review by Yakovenko shows that entropy production and the MEP are not limited to physical phenomena, but are evidence of a deeper numerical phenomenon that becomes apparent in physics, in economics, and elsewhere. [12]

But, the PMM is a closed mass/energy model, and real-world economies are open mass/energy systems far from equilibrium. Does the second law of thermodynamics apply to open systems? How? Recent papers propose that there is another still-poorly-understood law of thermodynamics describing entropy production for systems in a stationary state far from equilibrium. Sometimes called the Maximum Entropy Production Principle, or MEPP, it says that systems far from equilibrium may stabilize in a stationary state in which the rate of production of entropy is maximized (my paraphrase again of [13]). We find a remarkably similar concept in the writings of Odum and Hall called the "Maximum Power" concept. They argue that ecological systems reconfigure themselves by selecting constituent processes to consume energy at a maximum possible rate. [14] Since consumption of energy produces entropy, these two concepts appear to be identical. Odum and Hall have argued that the Maximum Power phenomenon applies to all natural mass/energy systems, including human economic systems. If you consider the vast flow of energy coming from fossil fuels over

the past 200 years, the MEPP has selected capitalism as the most effective economic process for entropy production, and created our modern world. López-Ruiz and others have made significant progress in defining entropy-like behavior in agent-based systems, for example at [15].

5   Some Testable Hypotheses

Research focused on the nature of simple sustainable economies is needed if we, the current cohort of the human race, are to truly implement economic policies that will craft a sustainable future for our children and our children's children. According to Boulanger and Bréchet the construction and study of agent-based models is the least expensive, fastest, and most effective way to undertake such research studies. [6] The following hypotheses are tabled herein to indicate potential future lines of investigation using ModEco and the PMM, or similar ABMs.

Definitions. These definitions are used in the following hypotheses:

Complete – Complete from top-to-bottom (economic and biophysical subsystems) and from end-to-end (complete mass and energy flows), in a cycle (closed systems) or from source to sink (open systems). It does not include front-to-back completeness (survival of other species). That aspect is not included in ModEco-based economies.

Conservative – Mass and energy must be absolutely conserved in all activities whether biophysical or commercial. The model may be closed or open with respect to energy and mass.

Simple – Having the same or similar biophysical subsystem as a ModEco-based economy, but possibly lacking one or some of the controls and features added to the baseline economic subsystem.

Sustainable – Able to run to one million cycles of the economic engine without collapse of the biophysical system and without exponentially climbing inflation.

Agent-based – In this context, the biophysical subsystem includes at least two types of mortal agents: farmers who own farms, and workers who are hired to work on the farms. All transfers of mass or energy between mortal agents, and all services rendered between mortal agents, must be reciprocated with cash.

Conforming Model Economy – In the hypotheses that follow, the words "conforming model economy" are meant to imply computer-based model economies which are complete, conservative, simple, sustainable, and agent-based, in the meaning of the words given above. One can assume that the PMM is a conforming model economy until proven otherwise, and so tests can be done using the PMM.

5.1   The Necessity and Sufficiency Hypothesis

Hypothesis #1: The PMM as found in ModEco is the most simple conforming model economy that can be constructed.

Discussion: The goal of this hypothesis is to focus attention on the questions of necessity and sufficiency of the various features of simple sustainable economies. The

goal is not to produce the smallest model with the least steps in the transition rule, or the least number of types of stores, or the least complexity in the code. The goal is to eliminate unneeded economic constraints or structure, and better understand the roles of those that are necessary. It is expected that this hypothesis will be easy to disprove, but instructive in the disproving.

Testability: To disprove the hypothesis, one needs to build a more simple conforming model economy. To be judged more simple, it must remove one of the complicating economic controls or mechanisms found in ModEco.

5.2 The Price/Value Hypothesis

Hypothesis #2: It is not possible to construct a conforming model economy in which all negotiated transactions allow for the possibility of profit and loss.

Discussion: The PMM does not allow for profit and loss. This hypothesis focuses attention on the mechanism by which agents set prices. In ModEco-based economies agents can base price quotes on intrinsic values, monetary values (weighted average cost), or market values (most recent price experienced). They consult the mean and standard deviation values in their appropriate pricing genes and produce a quote normally distributed about the base value. The PMM only works when the mean of all gene values is 1, the standard deviation of all gene values is 0, and the base price is intrinsic value. In other words, all prices are fixed at intrinsic value. Details of the ModEco pricing algorithms can be found in the design documents at [2].

Testability: To disprove this hypothesis, one needs to build a conforming model economy in which prices are set by the negotiating agents in some fashion, and one of the participating agents may make a profit on the deal.

5.3 The Intrinsic Value Hypothesis

Hypothesis #3: All conforming model economies are such that the biophysical needs of the agents either implicitly or explicitly control the price of food.

Discussion: From a biological point of view, we must consume energy and produce waste heat to live. The precepts of biophysical economics, and ecological science, would argue that this hypothesis has to be true. Can a conforming model economy be constructed in which this is not true?

Testability: To disprove this hypothesis a conforming model economy would have to price all goods and services in a fashion completely independent of metabolic needs, even while those metabolic needs control life functions.

5.4 The Central Authority Hypothesis

Hypothesis #4: Any conforming model economy must include immortal non-local agents performing the roles of the MMgr and EMgr.

Discussion: The focus of this hypothesis is to explore the aspects of these roles that make sustainability possible.

Testability: To disprove this hypothesis one needs to build a conforming model economy in which the functions of the central agents become part of the per-tick activities of the localized mortal agents, and the central agents are removed.

5.5 Social Justice Hypothesis

Hypothesis #5: In any conforming model economy, the majority of agents are poor. Subsidiary hypothesis A: These poor agents live shorter lives than their wealthy colleagues. Subsidiary hypothesis B: These poor agents experience energy throughput that is equal to or less than that of their more wealthy colleagues.

Discussion: It is believed that the distribution of wealth is a phenomenon with deep fundamental algorithmic roots, and the rich/poor gap is inescapable. It is further believed that poverty reduces access to participation in the economy, and reduces life span. This hypothesis, along with its two subsidiary hypotheses, tests those beliefs, and explores the relationship between sustainability and social or economic justice. This appears to be three hypotheses, but it is all of a piece. Can one build a conforming model economy in which some type of economic justice is emergent?

Testability: To disprove this hypothesis one needs to build a conforming model economy in which: (i) there is a normal distribution, or otherwise unbiased distribution, of wealth; and (ii) wealth is not correlated with energy throughput; and (iii) wealth is not correlated with length of life.

5.6 MEP and MEPP hypotheses

Discussion: Here are two hypotheses on the topic of entropy. This is territory that is not completely understood, on the leading edge of Econophysics and Biophysical Economics. I believe that it is very important to understand the nature of the MEPP in respect of economic systems, and that both of these hypotheses are false. However, the MEPP is currently not even well understood and not widely accepted as a proven phenomenon in thermodynamic systems, let alone economic systems or computer-based models of economic systems. These hypotheses presuppose the definition of entropy for a conforming model economy.

Hypothesis #6.1: A closed conforming model economy initialized in a non-equilibrium state does not exhibit the MEP as it evolves through transient states towards its equilibrium state.

Testability: To disprove this hypothesis one must: (i) define entropy for a conforming model economy; and (ii) build a closed conforming model economy that demonstrates the MEP as it evolves towards its equilibrium state.

Hypothesis #6.2: An open conforming model economy initialized in a non-equilibrium state does not exhibit the MEPP as it evolves towards its stationary state far from equilibrium.

Testability: To disprove this hypothesis one must: (i) define entropy for the conforming model economy; and (ii) build an open conforming model economy that demonstrates the MEPP as it evolves towards its stationary state far from equilibrium.

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