The 2015 AMMCS-CAIMS Congress
Waterloo, Ontario, Canada
June 7-12, 2015

The Equilibrium-Seeking Behaviour of a Very Simple Model of Wealth

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This material arises from a larger ongoing project which explores the dynamics of sustainable economies and seeks to identify some of the necessary and sufficient conditions for sustainability. Agent-based computer models (ABMs) do not just simulate such dynamics, but exhibit them, and so may prove to be a singularly useful testbed for such studies. This project has advanced in three overlapping streams of activity. First, it was inspired by capital exchange models described by Bennati, Drăgulescu and Yakovenko (see Refs [1, 2]). It has been shown that rising entropy clearly plays a role guiding those systems towards equilibrium. Scalas, Garibaldi and Donadio examined the mathematical properties of such models, but without reference to entropy (see Ref [3]). To better understand the role that entropy plays, a similar ABM was constructed with the goal of generating entropy-related data. Second, MS Excel has been used not only to analyse the output of that ABM, but also to construct and analyze a comprehensive union of all state spaces of all those models that differ only in the amount of initial total wealth endowment. Third, the results are translated into mathematical formulae. Analysis of the model's behaviour is ongoing, but the interim results are presented here.

The presentation starts with a description of the computer model, its origins, its basic functioning, and its relation to other similar models. The state space of the model is then described in the context of the comprehensive state space of which it is a part. With some arbitrariness, a formula for the entropy of a state is introduced, and an entropic index of a state is defined on the interval [0, 1]. A scatter plot of entropic indices of all possible states in the comprehensive state space is presented, displaying a curious pattern of apparent lines. Mathematical formulae for the apparent lines are presented, the significance of such lines is discussed, and the formula for the virtual enveloping curve of the comprehensive space is presented and discussed.

Then using an extremely small state space as an example, the set of all possible transitions is described for every state, the probability of state-to-state transition is computed for each transition pair, and the relationship between probability of transition and change in entropic index is explored. The source of the "arrow of time" that drives the model towards equilibrium is discussed in the context of edge effects (disallowed transitions) and asymmetric probabilities of transition between members of each transition pair. The weighted average entropic change is then calculated for each state, and the virtual equilibrium state is defined as that state for which the weighted average entropic change is 0.

Finally, using a somewhat larger state space, three characteristic aspects of the model are examined: the density of states is plotted against their entropic indices on the interval [0, 1]; the 'at equilibrium' density of occupancies of states is also so plotted; and the 'at equilibrium' tarry times are examined for consecutively re-visited states. Speculative mathematical descriptions of these phenomena are presented.

The presentation ends with a plea for continued research into such ABMs, with the goal of improving our understanding of the dynamics of such ABMs, and, ultimately, hopefully, of sustainable economic systems.

References

[1] E. Bennati, *Un metodo di simulazione statistica per l'analisi della distribuzione del reddito,* Rivista Internazionale di Scienze Economiche e Commerciali, 35, pp. 735-756 (1988).

[2] A. Drăgulescu and V. M. Yakovenko, *Statistical mechanics of money*, Eur. Phys. J. B., 17(4), pp. 723-729 (2000).

[3] E. Scalas, U. Garibaldi and S. Donadio, *Statistical equilibrium in simple exchange games I*, Eur. Phys. J. B., 53(2), pp. 267-272 (2006).